A] a) $Pr(\bar{A}) = 1 - Pr(A) = 1 - 0{,}5 = 0{,}5$, $Pr(\bar{B}) = 1 - 0{,}4 = 0{,}6$

b) $Pr(A \cap B) = Pr(A) \cdot Pr(B) = 0{,}5 \cdot 0{,}4 = 0{,}2$

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B) = 0{,}5 + 0{,}4 - 0{,}2 = 0{,}7$

B] a) $\int_0^2 \alpha \, x \, dx = 1 \Rightarrow \alpha \int_0^2 x \, dx = 1 \Rightarrow \alpha \left[\frac{x^2}{2}\right]_0^2 = 1 \Rightarrow 2\alpha = 1 \Rightarrow \boxed{\alpha = \frac{1}{2}}$

b) $\forall \, x \in (-\infty, 0)$ $F_X(x) = \int_{-\infty}^x 0 \, dt = 0$

Para $x \in [0,2]$ $F_X(x) = 0 + \int_0^x \frac{1}{2} \, dt = \frac{1}{2}\left[\frac{t^2}{2}\right]_0^x = \frac{1}{2}\left[\frac{x^2}{2} - \frac{0}{2}\right] = \frac{x^2}{4}$

Para $x \in [2, +\infty]$ $F_X(x) = 1 + \int_2^{+\infty} 0 \, dt = 1$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x^2/4 & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

c) $E(X) = \int_0^2 x \, f_X(x) dx = \int_0^2 \frac{x^2}{2} dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \frac{4}{3}$

$E(X^2) = \int_0^2 x^2 f_X(x) dx = \int_0^2 \frac{x^3}{2} dx = \frac{1}{2}\int_0^2 x^3 dx = \frac{1}{2}\left[\frac{x^4}{2}\right]_0^2 = 2$

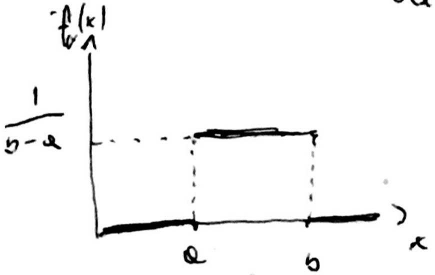
$\sigma^2(X) = E(X^2) - E^2(X) = \frac{2}{9} \simeq 0{,}222$ $\quad \sigma(X) = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3} \simeq 0{,}4714$

d) • $Pr(X < 1/2) = \int_0^{1/2} \frac{x}{2} dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^{1/2} = \frac{1}{16}$

• $Pr(1/2 < X < 1) = \int_{1/2}^1 \frac{x}{2} dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_{1/2}^1 = \frac{3}{16}$

• $Pr(X > 1) = Pr(1 < X < 2) = \int_1^2 \frac{x}{2} dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_1^2 = \frac{3}{4}$

D] a) $1 = \int_a^b c\,dx = c\int_a^b dx = c(b-a) \Rightarrow c = \frac{1}{b-a}$ DISTRIBUZIONE UNIFORME



b) $\mathbb{E}[X] = \int_a^b x \frac{1}{b-a}\,dx \Rightarrow \frac{1}{b-a}\,\frac{1}{2}x^2\Big|_a^b \Rightarrow \frac{1}{b-a}\,\frac{b^2-a^2}{2} \Rightarrow \frac{a+b}{2}$

c) $\mathbb{E}[X^2] = \frac{1}{b-a}\int_a^b x^2\,dx \Rightarrow \frac{1}{b-a}\,\frac{1}{3}x^3\Big|_a^b = \frac{b-a}{3(b-a)} = \frac{a^2+ab+b^2}{3}$

$\sigma(x) = \mathbb{E}(X^2) - \mathbb{E}^2(x) = \frac{a^2+ab+b^2}{3} - \frac{(a+b)^2}{4} = \frac{(b-a)^2}{12}$