DATA SCIENCE

Dynamic Model

Antonio VIOLI

antonio.violi@unirc.it

Introduction

- The Markowit's model is static and implements a buy and hold policy
- In real life investors may want to change their asset allocation as time goes on and new information becomes available
- This leads to dynamic models for asset allocation
- We present the deterministic version of the model

The dynamic model

- We consider a planning horizon divided into a number of elementary periods $t = 1, 2, \dots T$
- The aim is to create and manage a portfolio of assets so to maximize the wealth at the end of the planning horizon
- As time goes on, the investor may want to re–balance the current portfolio, by selling assets with worse performance and buying assets with better performance

The dynamic model

- We assume to consider N assets
- At each period t of the planning horizon, the investor must decide, for each asset i:
  - The amount of security to be purchased B_{it}
  - The amount of security to sell S_{it}
  - The amount of security to be maintained in the portfolio H_{it}

Model constraints: Physical Balance Constraints

- We have to model the dynamic evolution of the investment process
- Physical balance constraints

$$H_{it} = H_{it-1} + B_{it} - S_{it} \quad t = 1, \ldots, T-1, \quad i = 1, \ldots N$$

- For $t = 1$, H_{i0} represents some initial holding in asset i if any, otherwise we set the corresponding values to 0
- For $t = T$, different policies may be adopted. For example, we may impose to sell all the assets composing the portfolio at time $T-1$. In this case, we impose that $H_{iT} = 0$ and $B_{iT} = 0$.

Model constraints: Monetary Balance Constraints

We introduce the following parameters:

- P_{it} price of one unit of asset i at time t
- g transaction cost assumed to be proportional to negotiated amount
- L_t liability due in time t
  F_t cash to invest at time t

We assume that we may also invest in a risk-free asset that guarantees a given rate of return r_t
We denote by v_t the amount invested in the risk-free asset

Model constraints: Monetary Balance Constraints

Flow balance constraints ($t = 1, \ldots, T - 1$)

- inflow

$$(1 - g) \sum_{i=1}^{N} P_{it} S_{it} + F_t + (1 + r_t) v_{t-1}$$

- outflow

$$(1 + g) \sum_{i=1}^{N} P_{it} B_{it} + L_t + v_t$$

For $t = 1$ v_{t-1} is not defined

Model constraints

- We assume that at time T, all the assets in portfolio at time $T-1$ are sold.
- We define the wealth the end of the planning horizon

$$W_T = (1-g)\sum_{i=1}^{N} P_{iT} H_{iT-1} + (1+r_T)v_{T-1} + F_T - L_T$$

The objective function

- The aim of the model is to create and manage a portfolio that maximizes the wealth at the end of the planning horizon

$$\max z = W_T$$

Exercise

- Reformulate the Markowit's model by assuming that the decision variables x_i represent the holding in a given asset i
- By denoting with P_{i0} the price of asset i at time 0, the consistency constraint becomes ....
- The constraint on the expected performance becomes ...
- The objective function is ...