Impostare la risoluzione dei seguenti integrali
(stabilire se bisogna integrare per fili, per strati e/o applicare un opportuno cambio di coordinate)

Esempio

$$\iiint_\Omega \frac{xz}{x^2+y^2}\,dx\,dy\,dz \qquad \Omega=\{(x,y,z)\in\mathbb{R}^3:\ x\geq 0,\ y\geq 0,\ x^2+y^2\leq z\leq 1\}$$

Usiamo coordinate cilindriche

$$\begin{cases} x=\rho\cos\vartheta \\ y=\rho\sin\vartheta \\ z=z \end{cases} \qquad \begin{matrix}\cos\vartheta\geq 0\\ \sin\vartheta\geq 0\\ \rho^2\leq z\leq 1\end{matrix} \implies \begin{matrix}\vartheta\in[0,\frac{\pi}{2}]\\ \rho^2\leq z\leq 1\end{matrix} \implies \begin{matrix}\rho\in[0,1]\\ \vartheta\in[0,\frac{\pi}{2}]\\ \rho^2\leq z\leq 1\end{matrix}$$

$$|J|=\rho$$

$$I=\int_0^1\left(\int_0^{\frac{\pi}{2}}\left(\int_{\rho^2}^1 \frac{\rho\cos\vartheta\cdot z}{\rho^2}\cdot\rho\,dz\right)d\vartheta\right)d\rho=\int_0^1\left(\int_0^{\frac{\pi}{2}}\cos\vartheta\left(\int_{\rho^2}^1 z\,dz\right)d\vartheta\right)d\rho$$

1) $\iiint_\Omega x^2yz\,dx\,dy\,dz \qquad \Omega=\{(x,y,z)\in\mathbb{R}^3:\ 0\leq x\leq 1,\ 0\leq y\leq 1-x,\ 0\leq z\leq 1-x-y\}$

2) $\iiint_\Omega x\log y\,dx\,dy\,dz \qquad \Omega=\{(x,y,z)\in\mathbb{R}^3:\ 0\leq y\leq 2,\ x^2+z^2\leq y\}$

3) $\iiint_\Omega x\,dx\,dy\,dz \qquad \Omega=\{(x,y,z)\in\mathbb{R}^3:\ 1-x^2-y^2\leq z\leq 3,\ x^2+y^2\leq 1\}$

4) $\iiint_\Omega \frac{x}{z}\,dx\,dy\,dz \qquad \Omega=\{(x,y,z)\in\mathbb{R}^3:\ 1\leq x^2+y^2+z^2\leq 4,\ x\geq 0,\ y\geq 0,\ z\leq -\frac{1}{2}\}$

5) $\iint_\Omega \log(xy)\,dx\,dy \qquad \Omega=\{(x,y)\in\mathbb{R}^2:\ 1\leq x\leq 2,\ 1\leq xy\leq 3\}$

6) $\iint_\Omega \frac{x}{\log y}\,dx\,dy \qquad \Omega=\{(x,y)\in\mathbb{R}^2:\ y\geq 1,\ y\leq x^2\leq 4\}$

7) $\iint_\Omega xy\,dx\,dy \qquad \Omega=\{(x,y)\in\mathbb{R}^2:\ x\geq 0\ \ x^2+2y^2\leq 1\}$

8) $\iiint_{\Omega} (x^2+yz)\,dx\,dy\,dz \qquad \Omega = \{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2\leq 1,$
$\frac{1}{2}\leq z\leq x^2+y^2\}$

9) $\iiint_{\Omega} x\,\lg y\,dx\,dy\,dz \qquad \Omega = \{(x,y,z)\in\mathbb{R}^3 : y\geq 0,\ x^2+y^2\leq 2$
$1\leq z\leq 3\}$

10) $\iiint_{\Omega} x^2 z\,dx\,dy\,dz \qquad \Omega = \{(x,y,z)\in\mathbb{R}^3 : y\geq 0,\ y^2\leq x^2+z^2\leq y\}$